PMM U.S.S.R.,Vol.52,No.4,pp.461-466,1988
Printed in Great Britain

0021-8928/88 $10.00+0.00

CONICAL ROTATIONAL FLOW INDUCED BY TANGENTIAL STRESSES ON FREE PLANE SURFACE*

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An exact formulation is used to solve the problem of the effect of rotating tangential stresses of prescribed form, exerted on a free plane surface of a viscous fluid. The qualitative structure of the flow which appears within the fluid is studied. The use of the results to calculate the upwelling caused by hurricanes outside the zone of the strongest winds, yields real numerical values. The motion of the fluid studied here refers to the class of selfsimilar conical flows, various types of which were studied in /1-7/.

1. Formulation of the problem. We consider a half-space filled with a viscous incompressible fluid bounded by a horizontal free surface. There is no force of gravity. We introduce a spherical coordinate system R, θ, φ with origin on the free surface. The polar axis $\theta=0$ is perpendicular to the surface and directed towards the fluid. Tangential stresses are specified on the free surface. They have a rotational component τ, only, which varies according to a prescribed law $\tau=\rho c^2/R^2$; ρ is the density of the fluid and c is a constant. The flow is stationary and rotationally symmetrical; u, w, v are the velocity components corresponding to R, θ, φ. The characteristic parameters of the problem are the kinematic velocity of the fluid ν, and the constant c, and they are of the same dimensions. Analysis using the theory of similitude /8/ shows that the problem is selfsimilar, and its solution can be sought in the form

$$u=\frac{c^2 J(x)}{\nu r},\quad w=\frac{c^2 F(x)}{\nu r},\quad v=\frac{c^2\Omega(x)}{\nu r} \qquad (1.1)$$

$$(x=\cos\theta,\ r=R\sin\theta)$$

The equation of continuity yields $J(x)=F'(x)\sin\theta$. After subsituting (1.1) and eliminating $J(x)$, the Navier-Stokes equations reduce to the form /4/

$$\begin{aligned}
&F^{\mathrm{IV}}(1-x^2)-4xF'''+2kFF'''+6kF'F''=-\frac{4k\Omega\Omega'}{(1-x^2)} \\
&\Omega''(1-x^2)+2kF\Omega'=0 \\
&-2n=2kF^2+2k\Omega^2+[2k(FF''+F'^2)+F'''(1-x^2)- \\
&\quad 2F''x](1-x^2);\ k=c^2/(2\nu^2)
\end{aligned} \qquad (1.2)$$

Here k is a dimensionless parameter of the problem, and the function $n(x)$ is connected with the pressure by the relation $P/\rho=c^2n(x)/r^2+B$; B is a constant. The first equation of (1.2) is obtained after eliminating the pressure.

On the free surface $(x=0)$, are formulate the conditions for zero leakage and set the tangential stresses. We require that the vertical and rotational velocity components be bounded on the axis of symmetry, and that there should be no sources or sinks. As a result we obtain

$$F(0)=F''(0)=0,\ \Omega(0)=-1 \qquad (1.3)$$

$$F'(1)<\infty,\ F(1)=\Omega(1)=0 \qquad (1.4)$$

It should be noted that when conditions (1.4) hold, the vertical **accelerations** are also bounded as $x\to 1$.

Thus we have reduced the problem in question to that of solving the system of first two equations of (1.2) with boundary conditions (1.3), (1.4). We can transform this system to a more convenient form /4/. Let us integrate the first equation three times. This yields

$$(1-x^2)F'+2xF+kF^2=-k\int_0^x dx\int_0^x dx\int_0^x\frac{4\Omega\Omega'\,dx}{(1-x^2)}+S_0+S_1x+S_2x^2 \qquad (1.5)$$

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*Prikl.Matem.Mekhan.,52,4,594-600,1988

Using (1.3) we obtain from (1.5)

$$S_0 = F'(0), \quad S_1 = 0, \quad S_2 = \frac{1}{2} F'''(0) + k[F'(0)]^2 + F'(0)$$

We transform the expression for S_2, using the method employed in /9/ to study the turbulent jets. We multiply the first equation of (1.2) by $(1-x^2)$ and integrate it once. This yields

$$(1-x^2)^2 F''' + k(1-x^2)[2F'^2 + 2FF''] + 4kxFF' - 2kF^2 = -2k\Omega^2 + S_3 \tag{1.6}$$

The boundedness of $F'(1)$ implies that $(1-x)F'' \to 0, (1-x)^2F''' \to 0$ as $x \to 1$ (otherwise the functions F'' and $(1-x)F'''$ cannot be integrated when $x = 1$). Considering (1.6) at the points $x = 1$ and 0 and taking into account (1.3) and (1.4), we obtain

$$S_3 = 0, \quad F'''(0) + 2kF'^2(0) + 2k\Omega^2(0) = 0$$

In this case the expression for S_2 will take the form $S_2 = F'(0) - k\Omega^2(0)$. Transforming the triple integral on the right-hand side of (1.5) into the single integral /4/ and substituting the expression for S_0, S_1, S_2, we reduce (1.5) to the form

$$(1-x^2)F' + 2xF + kF^2 = -k\int_0^x \frac{2(x-t)(1-xt)\Omega^2\,dt}{(1-t^2)^2} + (1+x^2)F'(0) \tag{1.7}$$

From (1.7) and (1.4) we obtain, as $x \to 1$,

$$F'(0) = k\int_0^1 \frac{\Omega^2\,dt}{(1+t)^2} \tag{1.8}$$

Let us substitute this relation into (1.7) and make the substitution $F(x) = (1-x^2)f(x)/k$. Then (1.7) and the second equation of (1.2) will finally yield the system

$$f' + f^2 = k^2G(x)/(1-x^2)^2, \quad k = c^2/(2\nu^2) \tag{1.9}$$
$$\Omega'' + 2f\Omega' = 0$$
$$G(x) = (1-x)^2\int_0^x \frac{(1+t^2)\Omega^2\,dt}{(1-t^2)^2} + (1+x^2)\int_x^1 \frac{\Omega^2\,dt}{(1+t)^2}$$

We solve Eqs.(1.9) in the segment $[0, 1]$, with boundary conditions

$$f(0) = 0, \quad \Omega'(0) = -1, \quad \Omega(1) = 0 \tag{1.10}$$

In the present paper we prove the existence of solutions of system (1.9) with conditions (1.10) in the class of functions continuous in the segment $[0, 1]$. We study the quantitative structure of the solutions. We shall be able to confirm, by direct verification, that the function $F(x)$ satisfies conditions (1.3) and (1.4).

2. The structure of the solutions. *Lemma 1.* $\Omega(x)$ is a monotonically decreasing function, $\Omega(x) \geqslant 0$.

Proof. Integrating the second equation of (1.9) we obtain

$$\Omega'(x) = -E^2(0, x) < 0, \quad E(t, x) = \exp\left(-\int_t^x f\,du\right)$$

Then, taking into account (1.10) we find that $\Omega \geqslant 0$.

Lemma 2. The following relations hold:

$$G(x) \geqslant 0, \quad G(0) > 0, \quad G'(0) = G'(1) = 0, \quad G'''(x) > 0$$

Proof. The inequality $G(x) \geqslant 0$ follows directly from the expression for $G(x)$. By virtue of Lemma 1, $\Omega(x) \not\equiv 0$. Then $G(0) > 0$. Relations $G'(0) = G'(1) = 0$ can be confirmed by differentiating $G(x)$, taking the condition $\Omega(1) = 0$ into account. Since $G'''(x) = -4\Omega\Omega'/(1-x^2)$, by virtue of Lemma 1 we have $G''' > 0$.

Corollary 1. The inequality $f(x) \geqslant 0$ holds.

Proof. Multiplying the first equation of (1.9) by an integrating factor and integrating, we obtain

$$f(x) = k^2 \int_0^x \frac{G(t)\,E(t, x)\,dt}{(1-t^2)^2}$$

Since $G(x) \geqslant 0$ (Lemma 2), we have $f(x) \geqslant 0$.

Corollary 2. The following inequalities hold in the interval $0 < x < 1$:

$$\Omega'' > 0,\ \Omega(0) < 1,\ \Psi(x) < \Omega(x) < \Omega(0)(1-x)$$

$$\Psi(x) = \begin{cases} \Omega(0) - x, & 0 \leqslant x \leqslant \Omega(0) \\ 0, & \Omega(0) \leqslant x \leqslant 1 \end{cases}$$

Proof. From the proof of Corollary 1 and Lemma 1 it follows that $f(x) > 0$, $\Omega' < 0$ in the interval $0 < x < 1$. Then the second equation of (1.9) will yield $\Omega'' > 0$. The remaining inequalities follow from the last inequality, boundary conditions (1.10) and Lemma 1.

Lemma 3. The function $G(x)$ satisfies the inequality

$$G(0)(1-x)^2 \leqslant G(x) \leqslant \Omega^2(0)(1-x)^2$$

Proof. Since $\Omega(x) \leqslant \Omega(0)(1-x)$ (Corollary 2), therefore

$$G(x) \leqslant (1-x)^2 \int_0^x \frac{(1+t^2)\,\Omega^2(0)\,dt}{(1+t)^2} + (1+x^2)\int_x^1 \frac{(1-t)^2\,\Omega^2(0)\,dt}{(1+t)^2} \leqslant \Omega^2(0)(1-x)^2$$

(the following inequalities are used: $(1+t^2) \leqslant (1+t)^2$ for $t \geqslant 0$, $(1+x^2) \leqslant (1+t)^2$ for $t \geqslant x \geqslant 0$). To prove the left inequality, we introduce the function $H(x) = G(x) - G(0)(1-x)^2$. Taking into account Lemma 2 we obtain

$$H(0) = H(1) = 0,\ H'(0) > 0,\ H'(1) = 0,\ H'''(x) > 0$$

The function H'' should, by virtue of the conditions given, be first negative, and then positive. Then from a graphical construction we find, taking the boundary conditions for H and H' into account, that $H \geqslant 0$.

Corollary 3. The following inequality holds:

$$G(x) \geqslant \Omega^3(0)(1-x)^2/12$$

Proof. Taking into account Corollary 2, we obtain

$$G(0) \geqslant \int_0^{\Omega(0)} \frac{(\Omega(0)-t)^2\,dt}{(1+t)^2} \geqslant \frac{\Omega^3(0)}{12}$$

The latter, together with Lemma 3, proves the following assertion:

Lemma 4. The function $f(x)$ satisfies the inequalities $0 \leqslant f(x) \leqslant k^2 x$.

Proof. From the first equation of (1.9), Lemma 3 and Corollary 2 it follows that $f'(x) \leqslant k^2\Omega^2(0) \leqslant k^2$, and this yields $f(x) \leqslant k^2 x$. The inequality $f(x) \geqslant 0$ was proved in Corollary 1.

Using the above assertions we find the direction of the fluid flow near the axis of symmetry and the free surface, and study the asymptotic behaviour (as $k \to \infty$) of the solutions.

From the definition of radial velocity it follows that

$$vuR/c^2 = (1-x^2)\,f'/k - 2xf/k = kG(x)/(1-x^2) - (1-x^2)\,f^2/k - 2xf/k$$

Then $vuR/c^2 \to kG(0) > 0$ as $x \to 0$, since $f(0) = 0$), $vuR/c^2 \to 2f/k < 0$ as $x \to 1$ (by virtue of Lemmas 2 and 4). Thus the fluid flows away from the centre of rotation along the free surface, and towards the free surface near the axis of symmetry.

In the course of investigating the asymptotic properties of solutions we shall show that a boundary layer is formed near the free surface as $k \to \infty$, and we shall obtain estimates for the values of the functions.

Lemma 5. The following inequality holds:

$$\Omega(0) > 1/(4k)^{1/3} \quad \text{for} \quad k \geqslant 1$$

Proof. The proof of Lemma 4 yields $f \leqslant k^2\Omega^2(0)\,x$. Then

$$\Omega(0) = \int_0^1 E^2(0, t)\,dt \geqslant \int_0^1 \exp(-k^2\Omega^2(0)\,t^2)\,dt$$

or $z \geqslant (\pi^{1/4}/2^{1/2})\, k^{1/2} \mathrm{erf}^{1/2}(z)$, where $z = k\Omega(0)$, erf is the error integral. Graphical construction shows that the inequality holds for all $z \geqslant z_*$ where z_* is the solution of the equation $z_* = (\pi^{1/4}/2^{1/2})\, k^{1/2} \mathrm{erf}^{1/2}(z_*)$. The latter has a unique solution by virtue of the monotonic decrease in the value of the derivative of the right-hand side in z_*. We can establish by differentiating that $k(z_*)$, and hence $z_*(k)$ are monotonically increasing functions. Direct computation can confirm that $z_*(1) > 1/2$, therefore

$$z_*(k) > (\pi^{1/4}/2^{1/2})\, k^{1/2}\, \mathrm{erf}^{1/2}(1/2) > k^{1/2}/2 \quad \text{when} \quad k \geqslant 1$$

Since $z > z_*$, therefore $z > k^{1/2}/2$ when $k \geqslant 1$ and the assertion of the lemma follows.

Below we shall show that a boundary layer forms on the free surface as $k \to \infty$, whose thickness decreases at least as fast as $1/k^{1/4}$.

Taking into account Corollary 3 and Lemma 5, we can write for $k \geqslant 1$

$$f'(x) \geqslant \varkappa^2 - f^2, \quad \varkappa = k^{1/4}/(8\sqrt{6})$$

We introduce the function $\varphi(x)$ as a solution of the equation

$$\varphi' = \varkappa^2 - \varphi^2, \quad \varphi(0) = 0$$

It has been shown /4, 5/ that

$$f \geqslant \varphi \tag{2.1}$$

and $\varphi(x)$ is a monotonically increasing function. From the equation for φ we find that $\varphi' < \varkappa^2$, $\varphi \leqslant \varkappa^2 x$, hence

$$\varphi' \geqslant \varkappa^2 - \varkappa^4 x^2 \tag{2.2}$$

Let us write $\delta = 1/(\sqrt{2}\varkappa) = 8\sqrt{3}/k^{1/4}$. Then the following relations follow for $x \leqslant \delta$ from (2.2): $\varphi' \geqslant \varkappa^2/2$, $\varphi \geqslant (\varkappa^2/2)\, x$. By virtue of the monotonic increase in $\varphi(x)$ and the inequality (2.1), we have

$$f(x) \geqslant \varkappa/(2\sqrt{2}) \quad \text{for} \quad x \geqslant \delta \tag{2.3}$$

Repeated integration of the second equation of (1.9) from δ to $x > \delta$ and substitution of (2.3) into it yields

$$\Omega(x) = -\Omega'(\delta) \int_x^1 E^2(\delta, t)\, dt \leqslant -2\Omega'(\delta)\sqrt{e}\, \delta\zeta(x), \tag{2.4}$$

$$\zeta(x) = \exp(-(1/2)\, x\delta^{-1}) - \exp(-(1/2)\, \delta^{-1})$$

According to Corollary 2, $\Omega'' > 0$, hence $-\Omega'(\delta) < \Omega(0)/\delta$. This, together with (2.4), yields

$$\Omega(x)/\Omega(0) \leqslant 2\sqrt{e}\zeta(x)$$

The last inequality establishes the existence of a boundary layer whose **thickness** tends to zero at least as fast as $\delta \sim 1/k^{1/4} \sim (\nu/c)^{1/2}$.

The lower bound was obtained for $\Omega(0)$ in Lemma 5. Let us find the upper bound. Since $\Omega'' > 0$ (Corollary 2), it follows that $-\Omega'(\delta) < -\Omega'(0) = 1$ (condition (1.10)). Taking this into account we obtain, from (2.4),

$$\Omega(\delta) < 2\delta \tag{2.5}$$

Since $\Omega'' > 0$, therefore $\Omega(0) < \Omega(\delta) + \delta$. This, together with (2.5), yields $\Omega(0) < 3\delta = 24\sqrt{3}/k^{1/4}$.

3. Existence of solutions. Let $C[0, 1]$ be a space of functions continuous in the segment $0 \leqslant x \leqslant 1$, with the metric $\rho(f, g) = \max|f(x) - g(x)|$. We know that $C[0, 1]$ is a complex normed (and hence locally convex) space. Therefore, we can use a corollary from the Schauder fixed point theorem to prove the existence of solutions.

Theorem. System (1.9) with boundary conditions (1.10) has at least one fixed point on the segment $[0, 1]$.

Proof. Let D be a set of functions $f \in C[0, 1]$, satisfying the inequalities $0 \leqslant f(x) \leqslant k^2 x$. It is clear that D is a closed convex set. Let us introduce the mapping $Uf = g$:

$$g' = k^2 \frac{G(x)}{(1 - x^2)^2} - g^2, \quad g(0) = 0 \tag{3.1}$$

$$G(x) = (1 - x)^2 \int_0^x \frac{(1 + t^2)\, \Omega^2\, dt}{(1 - t^2)^2} + (1 + x^2) \int_x^1 \frac{\Omega^2\, dt}{(1 + t)^2}$$

We find the function $\Omega(x)$ from the relation

$$\Omega(x) = \int_x^1 E^2(0, t)\, dt \tag{3.2}$$

With $\Omega(x)$ thus defined, we find that the second equation of (1.9) and boundary conditions (1.10) are all satisfied.

According to (3.2) $\forall f \in D$, and the inequality $\Omega(x) \leqslant (1-x)$ holds. Then $0 \leqslant G(x) \leqslant (1-x)^2$ (Lemma 3) follows from the expression for $G(x)$, and $g(x)$ satisfies the conditions of Lemma 4. Thus $\forall f \in D$ and $g = Uf$ and the following inequality holds:

$$0 \leqslant g \leqslant k^2 x \tag{3.3}$$

From (3.3) and the inequality $G(x) \leqslant (1-x)^2$ it follows that problem (3.1) has a unique solution continuous on $[0, 1]$. Therefore, taking into account (3.3) we have $g \in D$, i.e. $U(D) \subset D$.

The mapping U is continuous, since $\Omega(x)$ can be expressed analytically in terms of f, $G(x)$ in terms of $\Omega(x)$, and $g(x)$ as a solution of (3.1) depends continuously on $G(x)$. The precompactness of $U(D)$ in $C[0, 1]$ can be proved using Arzelà's theorem /10/. The uniform boundedness of $U(D)$ follows from (3.3). From (3.1), (3.3) and the inequality on $G(x)$ there follows the uniform boundedness of $g'(x)$. From this it follows that the set $U(D)$ is equicontinuous.

Thus the mapping U and the set D satisfy the conditions of the corollary of the Schauder theorem, and this implies that a fixed point f of the mapping U exists.

4. The results obained above can be used to study the flow which occurs in a fluid in the case when a tornado-like vortex exists above its surface /11-14/ (examples of such vortices include hurricanes, storms /11, 12/ and their experimental laboratory models /13, 14/). The rotational velocity component outside the nucleus of the vortex (outside the zone of strongest winds /11/) can be approximately described by the relation $v = A/r$, A is a constant and r is the distance along the axis of rotation. If the flow is turbulent, then the magnitude of the tangential stress on the subjacent surface in the azimuthal direction is given by the relation /15, 16/ $\tau = c_1\rho_1 v^2$, c_1 is a constant and ρ_1 is the air density. Then we have $\tau \approx \rho c^2/r^2$, $c^2 = c_1\rho_1 A^2/\rho$ for the region outside the nucleus (within the zones B and C, using the terminology of /17/). Thus the solutions in question can only be used in the region outside the nucleus of the vortex. Any distortions of the free surface are neglected. For example, in the case of hurricanes the maximum distortion does not exceed 1m, which is much smaller than all the characteristic dimensions and does not even exceed the wave height. We note that since a radial flow exists in the boundary layer of the air vortex, radial stresses may appear at the surface of the fluid, not compensated by the distortion of the free surface. The above effects are neglected in this paper.

Comparing the inertia and Coriolis forces we find that when the rate of fluid flow is 1 m/sec, the inertia forces are greater than the Coriolis forces at distances of up to 20 km from the centre. Thus in the case of tornados or laboratory vortices, the contribution of the inertial mechanism discussed here towards the appearance of motion of the fluid will be dominant, and in the case of hurricanes it may be noticeable.

In order to obtain an estimate of the effect of the inertial mechanism, we computed the flow induced by a vortex with parameters close to those of a hurricane of medium strength. The method of successive approximations /4/ was used. We put $k = 1$ and 5 (the turbulent Reynolds number $c/\nu \approx 1.4$ and 3.2; this order of magnitude is normally used in computing flows belonging to the class in question /4, 9/), $c_1 = 10^{-3}$ /16/, $A = 10^6$ m^2/sec, $\rho_1 = 1$ kg/m^3, and $\rho = 10^3$ m^2/sec. Then $c = 10^3$ m^2/sec. The value of A was calculated for a hurricane of medium strength /11/, ($r_0 = 2 \times 10^4$ m is the radius of the strongest winds and $v_0 = 50$ m/sec is the velocity at $r = r_0$).

This yields the following result. If $k = 5$, $R = 2 \times 10^4$ and 2×10^5 and the depth is $h = 100$ m, then $w \approx 0.03$ and 1.3×10^{-3} cm/sec. When $h = 10^3$ m, we have $w \approx 0.3$ and 1.3×10^{-2} cm/sec. At the surface of the fluid, for the same k and R, we have $v \approx 10$ and 2 cm/sec, $u \approx 6$ and 1.3 cm/sec.

The corresponding values for $k = 1$ are approximately 4 times smaller for w and u, and 1.5 times smaller for v.

The value of w at $R = 2 \times 10^4$ m and $h = 100$ m is of nearly the same order as the vertical velocity calculated /17/ for zone B, and at $R = 10^5$ m for zone C. Solving for w at $h = 10^3$ m we find that the meridional circulation caused by the vortex extends to considerable depth and is not limited to the upper layer. This agrees with the mesurements carried out in the wake of a tropical cyclone when changes in temperature of the ocean deviating from the normal values were observed at depths of at least 10^3 m /18/.

Thus, using the exact formulation we have studied a vortical flow initiated by rotational tangential stresses on a free surface of a viscous fluid. In calculating the upwelling caused by the hurricane outside the zone of strongest winds, we obtained real numerical values. Two phenomenological parameters (c_1 and k) were used in carrying out the computations.

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Translated by L.K.

PMM U.S.S.R.,Vol.52,No.4,pp.**466-473,1988**
Printed in Great Britain

0021-8928/88 $10.00+0.00

THE FLOW OF LIQUID DOWN AN INCLINED PLANE AT HIGH REYNOLDS NUMBERS*

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The stability of the flow of a layer of incompressible liquid with a free surface down an inclined plane under the force of gravity is investigated for the case of large Reynolds and Froude numbers. The amplitudes of the perturbations which lead to a non-linear problem are found. Problems with initial data are formulated, as well as the boundary value problems with conditions on a moving wall. It is shown that four characteristic zones appear in the field of flow in a transverse direction, changing successively from one to the next. It is noted that the proposed scheme enables one to study detached flows with recirculation zones. The scheme constructed here resembles in many ways the pattern of flow past a plate on which a boundary layer is developed with selfinduced pressure /1-4/.

1. Let a layer of incompressible viscous liquid flow down an inclined plane, making an angle θ with the horizontal, under the force of gravity directed vertically downwards. We shall assume that the unperturbed motion is steady-state motion, with velocity parallel to the inclined plane. We shall choose, as dimensional quantities, the parameters of the unperturbed motion: the velocity of the free boundary U_0, the height of the liquid layer H_0 and the density of the liquid ρ_0. Using them we introduce dimensionless dependent and independent variables. We shall use a Cartesian system of coordinates with the x' axis directed

*Prikl.Matem.Mekhan.,52,4,601-609,1988